BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B69 12012

SUBJECT: Effects of Landing Point Designator Errors on Landing the Lunar Module in a Circular Target Area - Case 310

DATE: December 2, 1969

FROM: K. P. Klaasen

ABSTRACT

The probability of successfully landing the lunar module (LM) in a circular or point target area can be increased by properly biasing the initial aim point and then using the landing point designator (LPD) to make a redesignation to the target. Errors involved in the operation of the LPD reduce the probability of landing in the target below that which would result from an exact redesignation. LPD errors arise from four major sources: IMU errors, LPD optical inaccuracy, absolute altitude errors, and errors due to redesignation quantization size. In any particular case, both the probability of success and the reduction in probability due to these LPD errors depend on the altitude of redesignation, the target area radius, the redesignation delta-V budget, and the LM automatic landing error ellipse. For a single redesignation, the probability of success is maximized by redesignating at the greatest permissible altitude. An exact redesignation made at this altitude will result in the maximum probability in any particular case. The effect of LPD errors is to reduce the probability of success below this maximum. These error effects are greatest for redesignations at the greatest permissible altitude, small target areas, large redesignation delta-V budgets, and small landing error ellipses. Redesignation procedures which involve multiple redesignations or a redesignation at the optimum altitude for each designated landing point are effective means of reducing LPD error effects. It is possible, however, that in some cases the LPD error effects will be small enough so that a redesignation procedure no more complex than a single redesignation at a prescribed altitude will be warranted.

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MEMORANDUM FOR FILE

INTRODUCTION

Earlier studies have considered the use of the landing point designator (LPD) to land the lunar module (LM) in a circular or point target area.[1,2] In these studies, the probability of landing in the target area was maximized by properly biasing the initial aim point and then making a single landing point redesignation to the target. This redesignation was assumed to be made without error. In practice, however, the errors introduced through the operation of the LPD are significant. The magnitude of these errors depends largely on the geometry of the LM descent trajectory and increases as the altitude of redesignation increases. However, since the redesignation capability in distance of a given redesignation delta-V budget also increases with altitude, there may be an optimum altitude at which the redesignation should be made in order to maximize the probability of successfully landing in the target area. In addition, a second redesignation made later in the descent can compensate for errors arising from the initial redesignation and thus increase the probability of landing in the target area.

REDESIGNATION ASSUMPTIONS

It is desired to find the method of using the LPD to make one or more landing point redesignations such that the probability of landing within a given circular area is maximized.

It is assumed that:

1. Only left and/or downrange redesignations are acceptable.

2. Redesignation capability in distance is determined by the redesignation delta-V budget and the altitude at which the redesignation is made as shown in Figures 1a and b.[3]

3. All redesignations are effected instantaneously.

4. All dispersions are normally distributed.

5. The descent trajectory for the Apollo 11 mission is used.

6. Redesignations above an altitude of 5,000 ft. are not permitted.

SINGLE REDESIGNATION WITH LPD ERRORS

The method of using the LPD to maximize the probability of landing the LM within a circular area using a single, exact redesignation at a given altitude is presented in Reference 2. For crossrange and downrange redesignation capabilities ρ_y and ρ_x and a target area T of radius Δ, any eventual landing point which is designated by the LPD to be within the area R, shown in Figure 2, will allow a redesignation to the target to be made. R is assumed to be bounded by a quarter ellipse of semi-axes $\rho_x + \Delta$ and $\rho_y + \Delta$. The initial aim point O, with coordinates (μ_x, μ_y), is biased uprange and to the right of the target so that the probability of landing in R, given by

$$P = \frac{1}{2\pi\sigma_x\sigma_y} \iint_R \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] dx\, dy$$

where σ_x and σ_y are the downrange and crossrange standard deviations of the LM automatic landing error ellipse, is maximized.

In reality, however, the operation of the LPD in making a landing point redesignation involves some degree of error. Thus, it is not true that for any designated landing point within R, that is, within the redesignation capability, the probability of successfully redesignating to the target is 100%. For each designated landing point A, a redesignation to T will be subject to some downrange and crossrange LPD errors defined by standard deviations σ_x' and σ_y', and the probability of successfully redesignating to T will be somewhat less than 100%.

In order to find the overall probability of reaching T, every possible point A must be considered. Given the initial aim point O and the standard deviations of the automatic landing error ellipse, σ_x and σ_y, the probability of landing in a given square area about A can be determined by integrating the two dimensional probability function defined by σ'_x and σ'_y over area A. For every point A, the optimum redesignation is made to that point B which is within the redesignation capability of the LM at the given redesignation altitude and which maximizes the probability of landing in T. This procedure is illustrated in Figure 3. Thus, for every point A, the probability of successfully redesignating to T can be determined by integrating the probability function defined by standard deviations σ'_x and σ'_y over T with the aim point coordinates being those of the optimum redesignation point B. The probability of landing in T for an originally designated landing point A then is given by the product of the probability of landing in area A times the probability of successfully redesignating to T from A. The overall probability of landing in T is then given by the summation of the probability of landing in T for each A over all A. The initial aim point O is chosen so as to maximize this overall probability.

SOURCE OF LPD ERRORS

The error ellipse associated with redesignation to a particular point using the LPD and defined by standard deviations σ'_x and σ'_y arises from four main sources. These error sources are IMU errors, LPD optical inaccuracies, absolute altitude errors, and errors due to redesignation quantization size.

LM descent simulations incorporating 3σ IMU errors show that inaccuracies during the relatively short time between hi-gate and touchdown are minor.[4]

The 3σ accuracy in optically determining the eventual landing point of the LM using the LPD has been calculated to be about $\pm 1.0°$.[5] These errors arise primarily from an inability to read the scribed scale on the LM window accurately.

Figure 4 shows the altitude error effect on LPD downrange accuracy due to terrain height difference between the landing site and the current LM position.[6] Notice that absolute altitude errors produce no crossrange landing site deviations. The LM landing radar weighting function helps to minimize the effects of short term altitude variations of the lunar terrain such as isolated peaks or craters. Altitude errors are mainly a result of a long term, general slope of the lunar surface rather than steep, localized slopes. The 3σ terrain slope of the lunar surface is assumed to be 1° for the primary Apollo landing sites. Thus, the 3σ altitude error is assumed to be given by

$$E_{alt} = (\text{range to go}) \times (\tan 1°)$$

The 3σ downrange error due to altitude errors is given by

$$E_{downrange} = \frac{E_{alt}}{\tan \alpha}$$

where α is the flight path angle of the LM descent trajectory. Later landing sites might possibly have a steeper slope along the approach path. This approximation would then have to be revised. Landing radar inaccuracy would produce the same downrange error effect as an altitude variation; however, the effects of landing radar errors are negligible when compared to even a 1° lunar slope.

In making a redesignation to a specific point, errors also arise from the fact that, due to LPD pulse size, redesignations can be made only in increments of 1/2° downrange and 2° crossrange. Thus there exist possible errors up to 1/4° downrange and 1° crossrange for a particular redesignation at a given altitude. These errors are not normally distributed about the desired aim point. However, in order to combine errors due to this source with those from the other LPD error sources, they were approximated by a normal distribution curve with 3σ equal to 1/4° downrange and 1° crossrange. A total LPD 3σ error ellipse for left and downrange redesignations can then be obtained by taking the root - sum - square of the errors due to each of the four error sources. Values of σ_x' and σ_y' along with their components from each error source in units of feet on the surface of the moon are plotted as a function of the altitude of redesignations in Figure 5.

PROBABILITY REDUCTION DUE TO LPD ERRORS

Maximum Possible Probability

A single redesignation without error made at the highest allowable altitude represents the upper bound on the probability of successfully landing in the target area for any target radius Δ and redesignation delta-V. This fact can be illustrated by a re-examination of Figure 2. For a single redesignation made at the highest allowable altitude the area R is a maximum since the redesignation capability is greatest. For an exact redesignation, any designated landing point within R allows a redesignation to the target to be made with 100% probability of success. The probability of reaching T with an exact redesignation then is simply the probability of landing in R without a redesignation. The initial aim point O is chosen so that the probability of landing in R is a maximum. If redesignation errors are introduced through the LPD operation, the overall probability of reaching T must necessarily be reduced.

The errors in redesignation limit the area in R from which a redesignation to T can be considered 100% successful to a smaller region near T as shown in Figure 6. Various changes in the redesignation procedure such as redesignating at the optimum altitude for each designated landing point A rather than at some preselected altitude, biasing the initial aim point differently, or using multiple redesignations to correct for previous errors can be implemented in order to reduce the effects of LPD errors. These procedures can increase the area of 100% successful redesignations. However, this area can never exceed that of R since all the procedures designed to reduce LPD error effects depend either on using less than the entire redesignation delta-V budget at the earliest opportunity, in which case it would be impossible to redesignate to a point in T from a designated point A on or near the boundary of R, or on biasing the initial aim point to a point other than that which maximizes the probability of landing in R.

Figures 1a, 1b and 5 show that both redesignation capability and LPD errors increase with the altitude of redesignation. The increase in redesignation capability tends to increase the probability of success while the increase in LPD errors tends to decrease it. In order to determine whether there exists an optimum altitude at which the redesignation should be made, the probability of landing in the target area was calculated for redesignations at different altitudes for several landing target area radii and several redesignation delta-V budgets using the Apollo 11 landing error ellipse (σ_x = 6481 ft., σ_y = 2836 ft.). The resulting probabilities are given in Figure 7 along with the probabilities for an exact redesignation. For all cases, the probability of success increased with altitude up to the limit of 5,000 ft. So for a single redesignation at a prescribed altitude, the optimum altitude of redesignation within the allowable limits is 5,000 ft. both for exact redesignations and for those including LPD errors.

Cases of Greatest LPD Error Effects

The difference between the probability curve for an exact redesignation and the probability curve for a redesignation including LPD errors as shown in Figure 7 represents the reduction in the probability of successfully landing in T due to LPD errors for a single redesignation at a prescribed altitude. The magnitude of the probability loss due to LPD errors is seen to increase with altitude and to decrease as Δ increases until for Δ greater than about 10,000 ft. the loss is negligible. This probability loss indicates those cases in which more complex redesignation procedures designed to reduce LPD error effects could bring about significant benefits in the form of increased overall probability of successfully landing in the target area. Making a redesignation at the optimum altitude for each designated landing point A or using multiple redesignations will give probabilities greater than those of a single redesignation at a prescribed altitude yet less than the maximum probability obtained from an exact redesignation at the greatest permissible altitude. Thus, these more complex procedures will be profitable only when the probability loss due to LPD errors is significant.

The magnitude of LPD error effects depends on the particular combination of landing target radius, redesignation delta-V, σ_x, σ_y, and the altitude of redesignation for any given case. Since LPD error effects increase with the altitude of redesignation, in determining the changes in LPD error effects due to the other parameters, redesignations were assumed to be made at 5,000 ft., the maximum allowable altitude. In Figures 8a, b, and c, probability curves are plotted for both exact redesignations and those including LPD error as functions of Δ, redesignation delta-V, and σ_x or σ_y for different combinations of the remaining parameters. The magnitude of the loss increases for decreasing Δ, and for Δ less than about 1,000 ft., the increase in LPD error effects is very marked. Variations in the size of the redesignation delta-V budget or the size or shape of the LM landing error ellipse do not affect the magnitude of the probability loss greatly except in those cases where Δ is less than 1,000 ft. The tendency is for the magnitude of the probability loss to increase as redesignation delta-V increases or the landing error ellipse decreases. This trend reaches a point of diminishing returns, however, as the value of the probability becomes high.

Absolute probability differences are not always meaningful, however, since a given difference in magnitude is much more significant at a high probability than at a low one. Thus, another measure of LPD error effects, D, was derived and is given by

$$D = \frac{P_{\text{w/o errors}} - P_{\text{with errors}}}{1.0 - P_{\text{with errors}}}$$

where P represents probability. In Figures 9a, b, and c, D is plotted vs. Δ, redesignation delta-V, and σ_x or σ_y for the cases plotted in Figures 8. Using this measure it is apparent that LPD error effects become more significant as redesignation delta-V increases and the landing error ellipse decreases. Figure 9a shows again that LPD error effects become much more significant for Δ less than 1,000 ft.

REDUCTION OF LPD ERROR EFFECTS

More Efficient Redesignation Procedures

The probability loss due to LPD error effects which was previously calculated was based on the difference between a single, exact redesignation and a single redesignation including LPD errors made at a given altitude. In the presence of LPD errors, a redesignation procedure in which the redesignation is made at a given, preselected altitude is not the most efficient procedure. In this case, no consideration is given to the location of the actual designated landing point at the time of redesignation. For designated landing points close to the desired target point, a redesignation to the desired point at 5,000 ft. will not use the budgeted redesignation delta-V to its greatest advantage. A more efficient procedure would be either to wait and redesignate at a lower altitude where LPD errors are smaller or to use only a part of the redesignation delta-V, saving the rest to use in making later redesignations to correct any errors which have arisen.

Redesignation at Altitude which Maximizes Probability

Figure 10 shows the proper altitude of redesignation for designated landing points distributed over the surface, assuming a redesignation procedure in which a single redesignation is made at that altitude for which the probability of landing in T is a maximum for each designated landing point. In this case, redesignation delta-V = 400 fps., Δ = 500 ft., σ_x = 6481 ft., σ_y = 2836 ft. Note that as designated landing points move further away from T the optimum altitude of redesignation increases up to the maximum of 5,000 ft. In general, redesignations are directed as close to the center of the target area as possible within the redesignation capability, and the optimum altitude of redesignation is approximately that at which a redesignation to the center of T requires use of the entire redesignation delta-V budget.

Double Redesignations

A double redesignation procedure was also examined. In this case redesignations were made at altitudes of 5,000 ft. and 1,000 ft. At 5,000 ft. there is a unique redesignation procedure for every designated landing point A which will maximize the probability of landing in T. This procedure is defined by the direction and magnitude of the redesignation to be made. Let K be the fraction of the redesignation delta-V budget to be used in this first redesignation. A new aim point B which is within the redesignation capability of the delta-V to be used at 5,000 ft. should be chosen so that the probability of landing in the area R defined by the radius of the landing target and the redesignation capability of the remaining redesignation delta-V at an altitude of 1,000 ft. is maximized. This procedure is illustrated in Figure 11. The redesignation at 1,000 ft. is directed as close to the center of T as is possible using only left and downrange redesignations. This final redesignation is assumed to be exact since the 3σ error involved in LPD operations at this altitude is only about 325 ft. downrange and 90 ft. crossrange. This error is assumed to be less than the translational capability available during the manually controlled hover period immediately before touchdown. For every A there is an optimum value of K and a new aim point associated with that K such that the probability of reaching T is maximized. The initial aim point O is chosen to maximize the overall probability of landing in T for all A.

Figure 12 shows the optimum redesignation procedure for designated landing points distributed over the surface. The arrows indicate the point to which the initial redesignation should be made. In this case, redesignation delta-V = 30 fps, Δ = 1,000 ft., σ_x = 6481 ft., σ_y = 2836 ft. Note that as designated landing points move further away from T, the fraction of redesignation delta-V to be used in the first redesignation becomes greater. Redesignations should be made in the general direction of the center of the target area with somewhat more emphasis on redesignation in the direction of greatest capability, uprange for redesignation delta-V budgets greater than about 200 fps and crossrange for smaller delta-V budgets.

Evaluation of Redesignation Procedures

The table presented in Figure 13 gives probability values for a single redesignation without error made at 5,000 ft., a single redesignation with LPD errors made at 5,000 ft., a single redesignation with LPD errors made at the optimum altitude for each designated landing point, and a double redesignation with LPD errors. Several combinations of Δ, redesignation delta-V, σ_x, and σ_y were examined. Note that the double redesignation and the single redesignation at the optimum altitude both give better probability of success than a single redesignation at a given altitude since they both lessen LPD error effects somewhat. However, neither procedure gives a probability greater than the single exact redesignation which represents the upper bound of probabilities in each case. It appears also that the double redesignation procedure corrects for LPD errors very well and gives probabilities close to those of an exact redesignation. It seems reasonable to expect that a procedure involving three or four redesignations would raise the probability even closer to that of an exact redesignation at the cost of increased crew task loading.

CONCLUSIONS

Errors inherent in the operation of the LPD reduce the probability of successfully landing in a circular target area using landing point redesignation. Redesignation procedures more complex than just a single redesignation made at a prescribed altitude may be used to help compensate for these errors. Multiple redesignations and redesignations made at the optimum altitude for each designated landing point are effective means of reducing LPD error effects. These more complex procedures will be worthwhile, however, only when the probability loss due to LPD errors is significant. The effect of LPD errors is the greatest for cases in which the landing target area is small, the automatic landing error ellipse is small, and the redesignation delta-V budget is large. This is not to say that small landing error ellipses and large redesignation delta-V budgets should be avoided, since it is in these cases that the probability of success is greatest. Rather, it is in these cases that a more complex redesignation procedure is most likely to be warranted in order to compensate for increased LPD error effects. It is also possible, however, that there are cases in which the error effects are small enough that a redesignation procedure more complex than a single redesignation at a given altitude would not be worthwhile.

K. P. Klaasen

2013-KPK-jab K. P. Klaasen

Attachments:
References
Figures 1 through 13

REFERENCES

1. Suddath, J. H. and R. L. Saldana, "Use of the Landing Point Designator to Land the Lunar Module to a Given Target", MSC Internal Note 67-EG-32, November 14, 1967.

2. Klaasen, K. P., "Use of the Landing Point Designator to Land the Lunar Module in a Circular Target Area", Case 310, Bellcomm Memorandum for File, B69 09039, September 16, 1969.

3. LaPiana, F., "Preliminary Results for LPD Pulse Costs and Effects", preparation for Bellcomm Memorandum for File, July 1969.

4. LaPiana, F., "Touchdown Position Deviations Due to LM PGNCS IMU Error Sources", Case 310, Bellcomm Memorandum for File, B69 06093, June 24, 1969.

5. Hoekstra, T. B., "Presentations on LM Descent at 8:30 APO Staff Meeting of June 10, 12 and 16, 1969", Case 310, Bellcomm Memorandum for File, B69 06072, June 23, 1969.

6. Cheatham, D. C. and D. E. Steele, "Analysis of Landing Point Designator Operation", MSC Internal Note 65-EG-52, December 9, 1965.

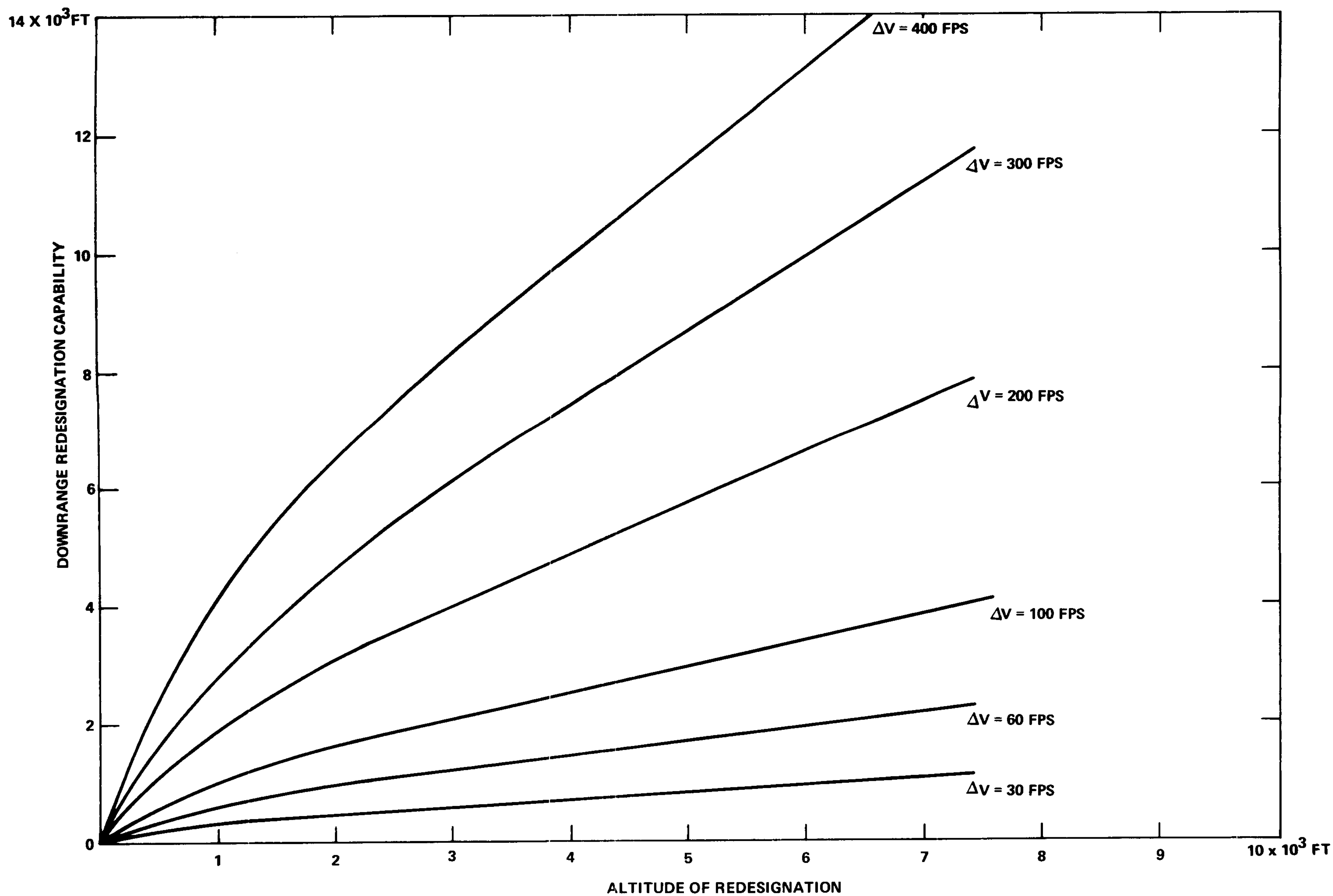


FIGURE 1A - DOWNRANGE REDESIGNATION CAPABILITY

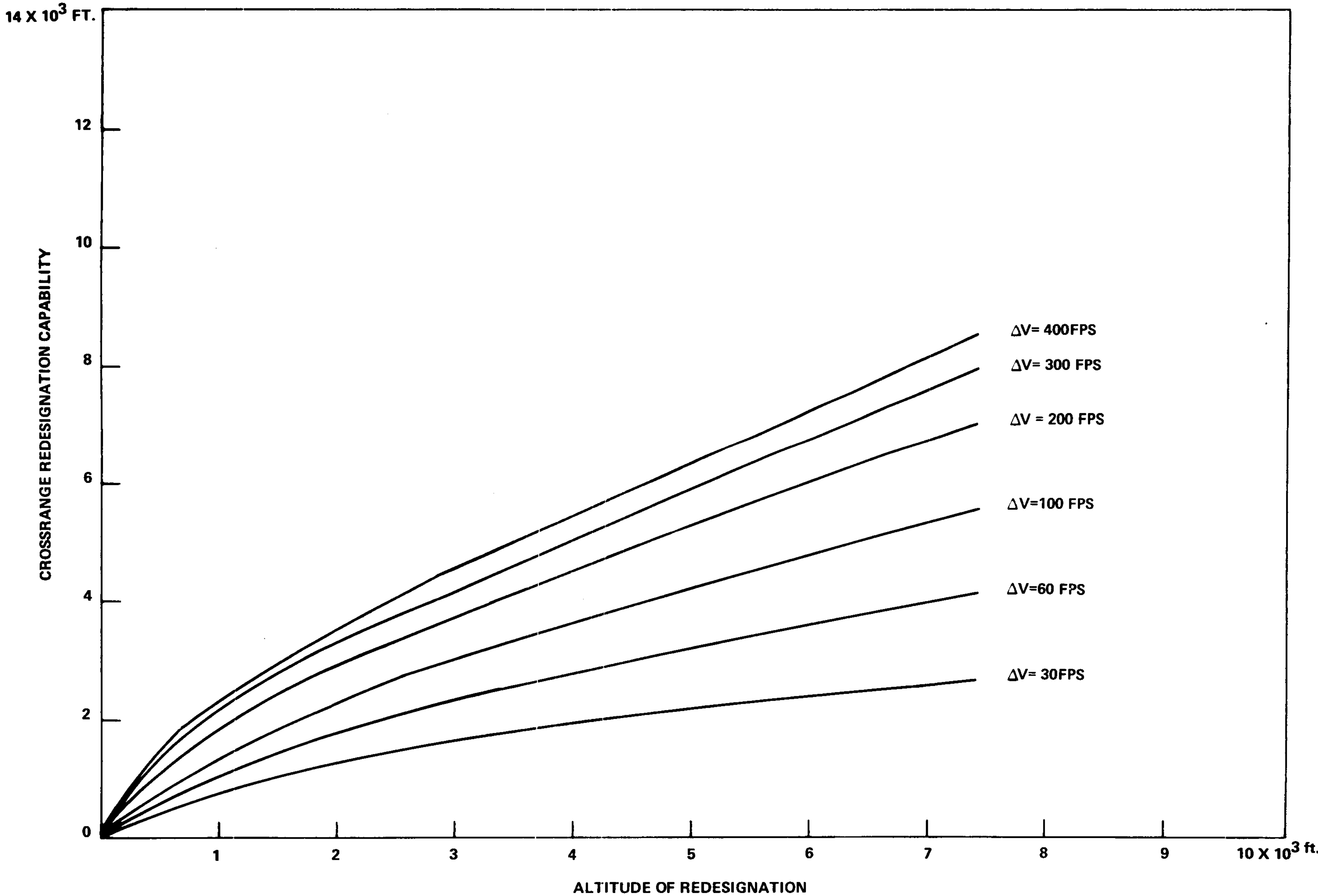


FIGURE 1B- CROSSRANGE REDESIGNATION CAPABILITY

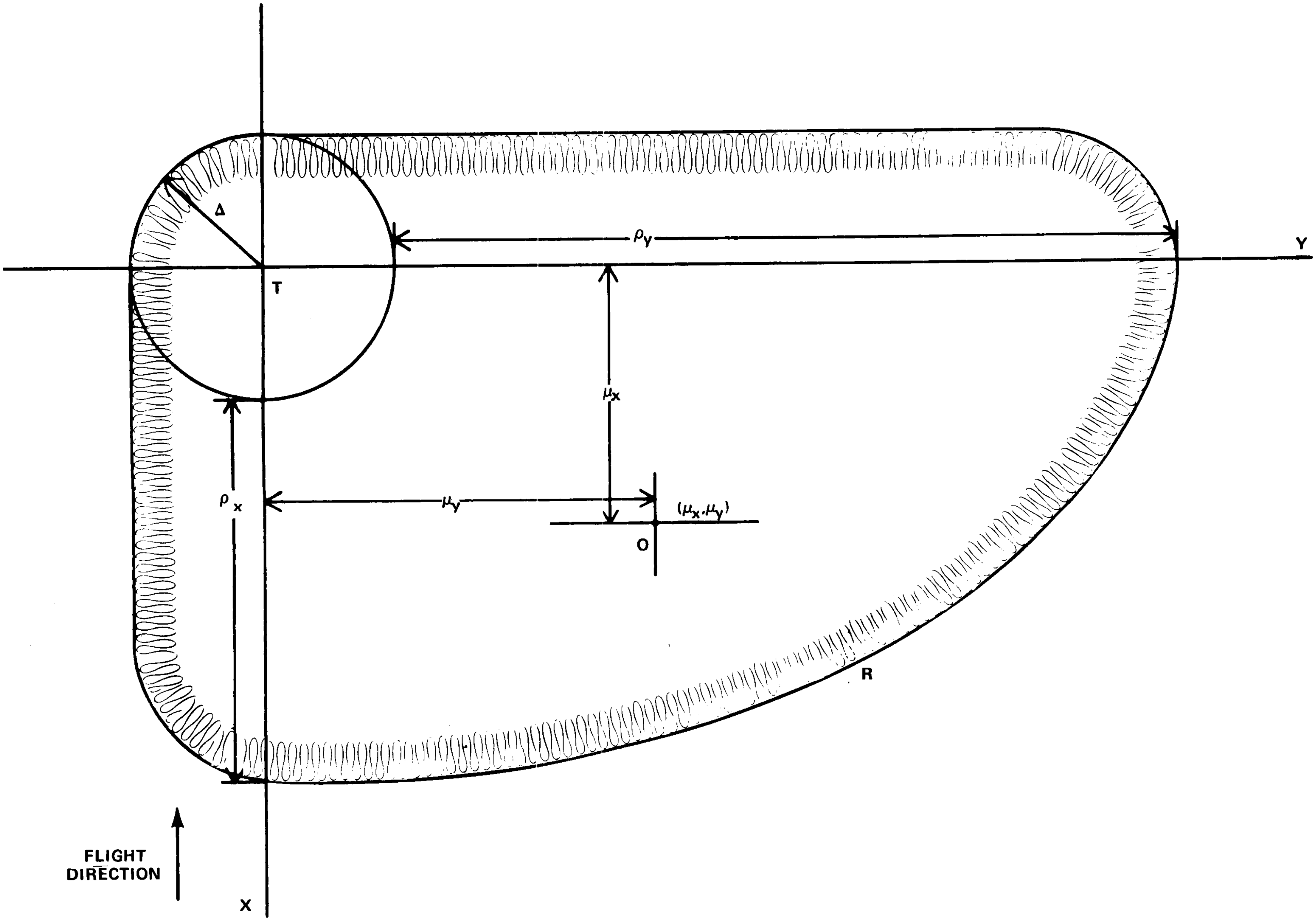
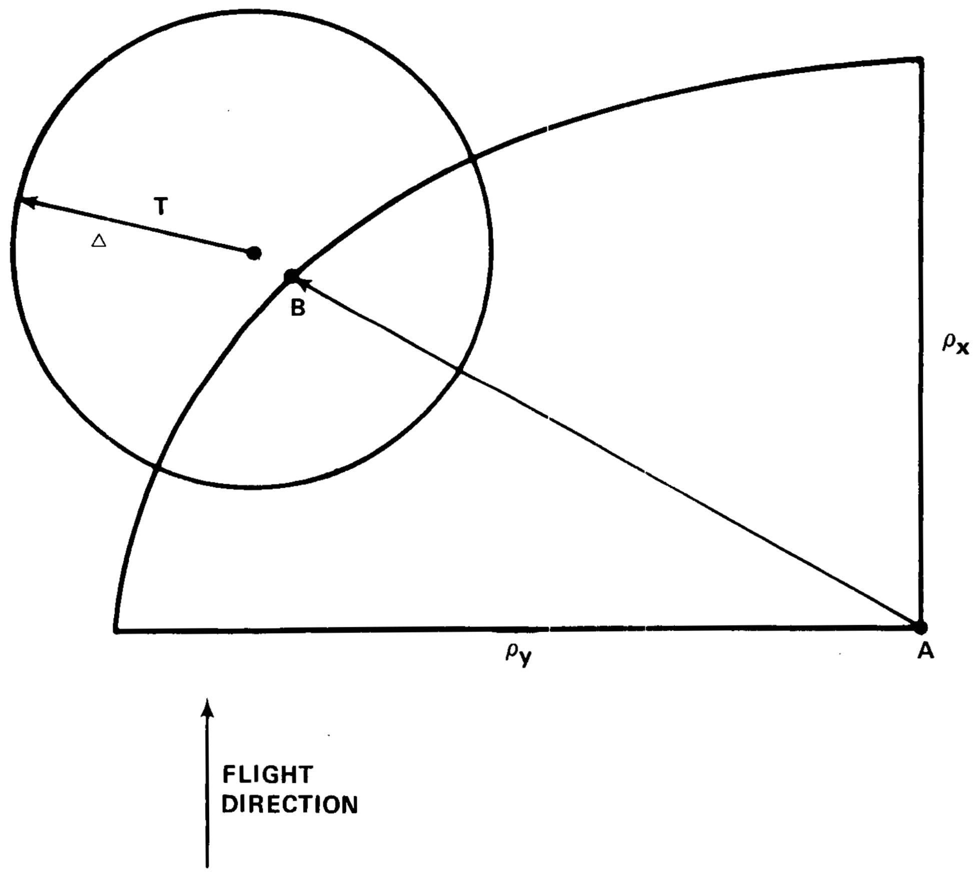


FIGURE 2 - TWO DIMENSIONAL REDESIGNATION TO TARGET OF RADIUS Δ



ρ_x = DOWNRANGE REDESIGNATION CAPABILITY

ρ_y = CROSS RANGE REDESIGNATION CAPABILITY

FIGURE 3 – REDESIGNATION PROCEDURE WHEN LPD ERRORS ARE INCLUDED

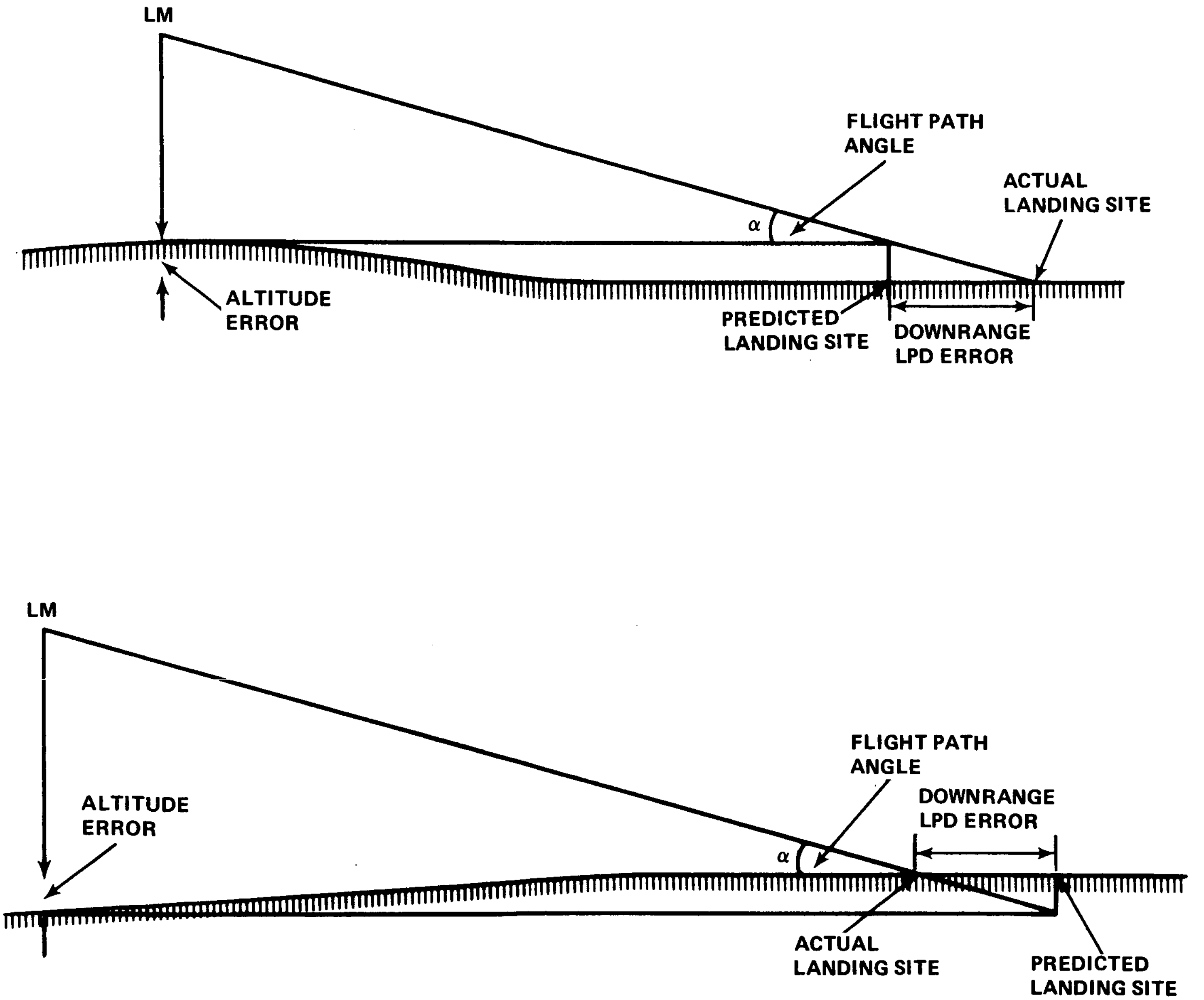


FIGURE 4 – ALTITUDE ERROR EFFECT ON LPD DOWNRANGE ACCURACY

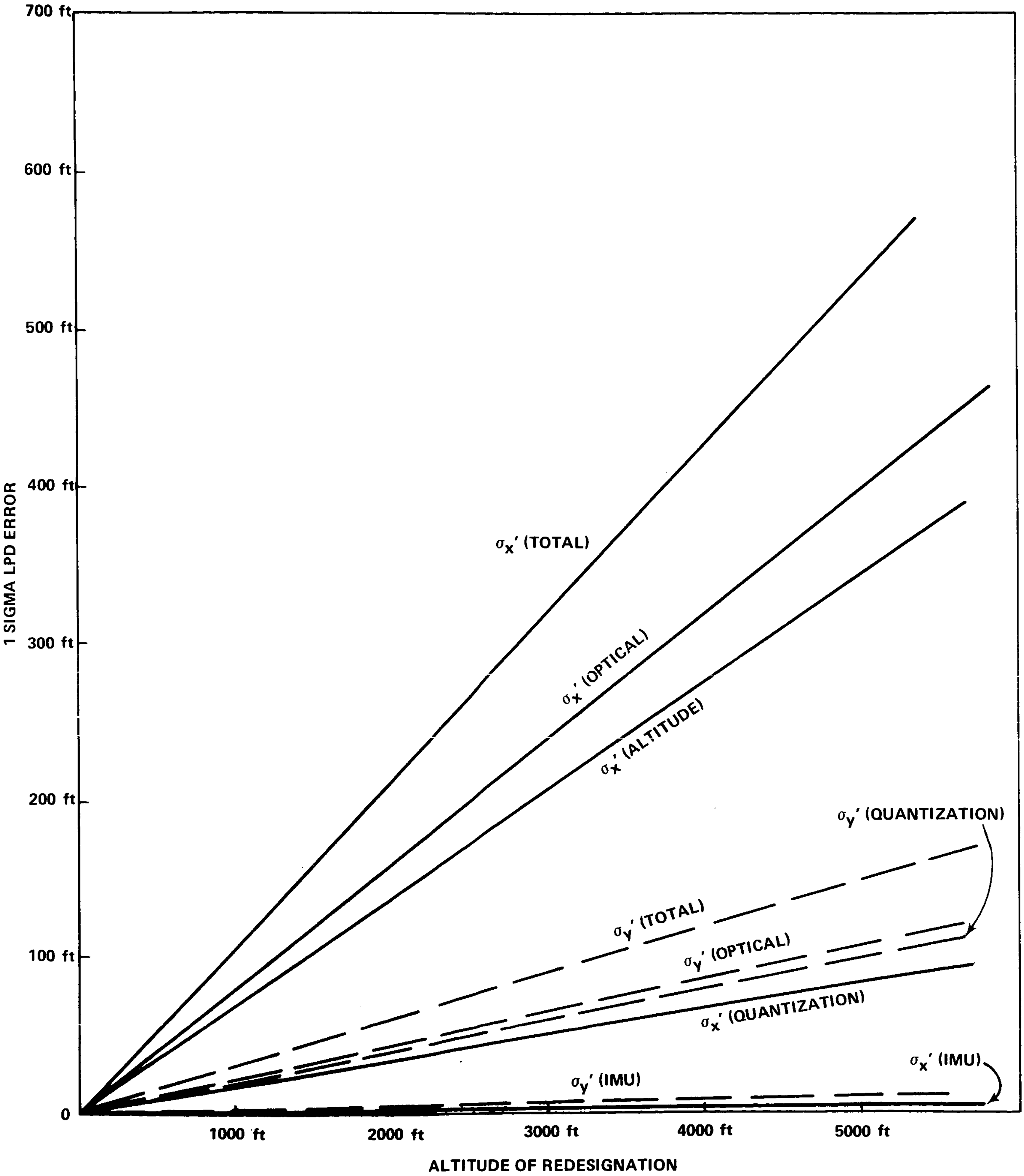


FIGURE 5 – SEMI-AXES OF DOWNRANGE AND CROSSRANGE LPD ERROR (1 SIGMA)

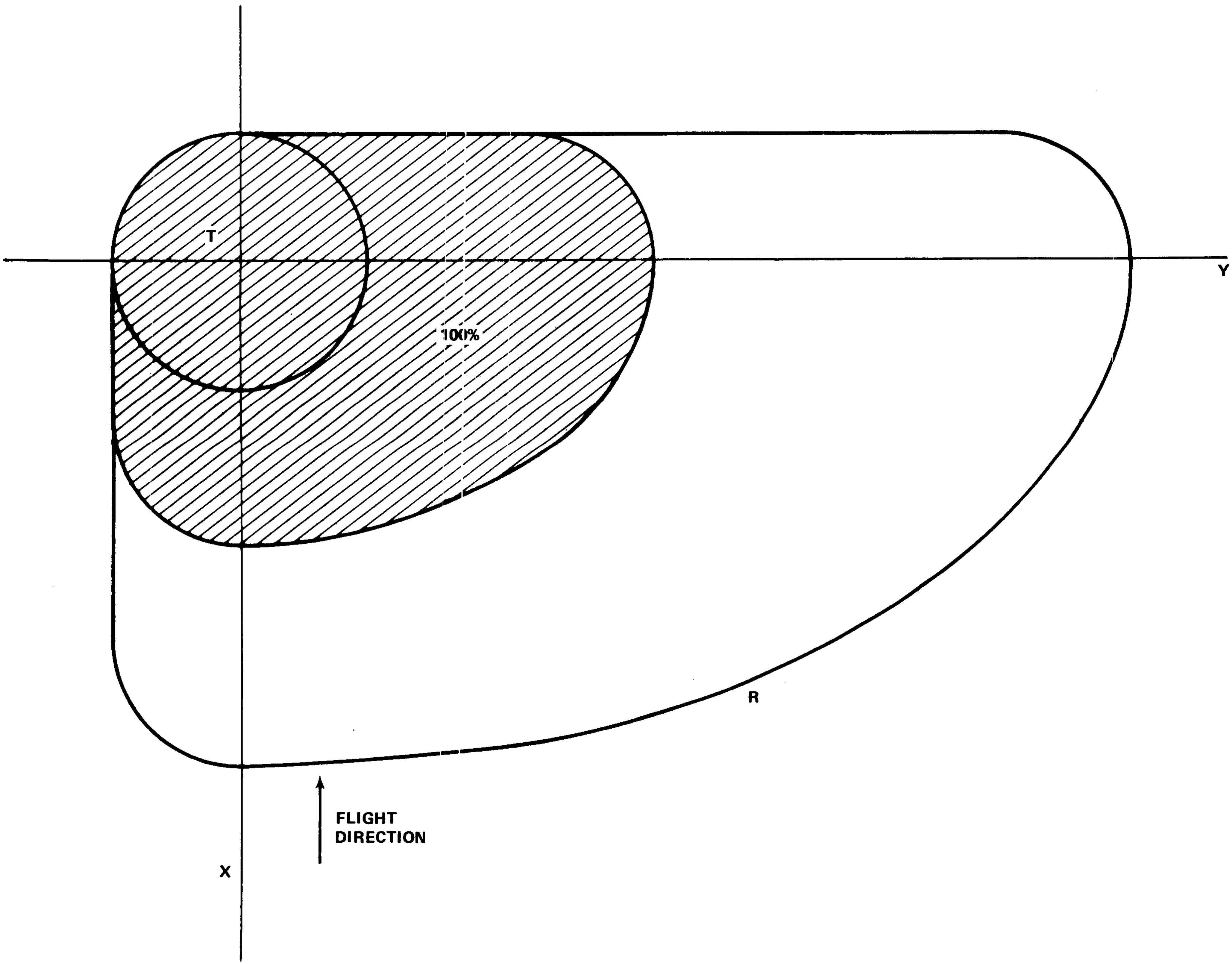


FIGURE 6 – AREA OF 100% SUCCESSFUL REDEGINATIONS TO T WHEN LPD ERRORS ARE INCLUDED

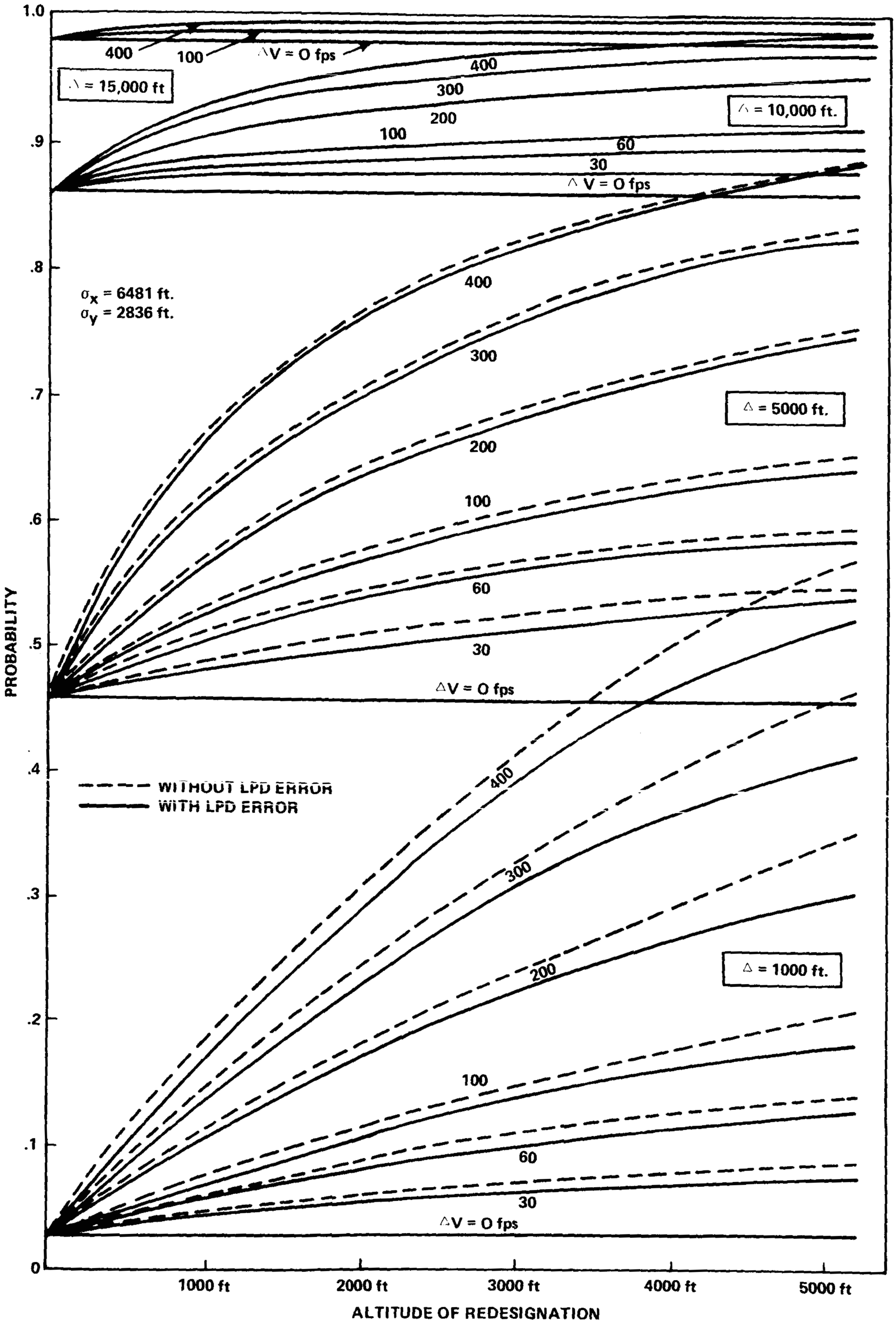


FIGURE 7 – SINGLE REDESIGNATION PROBABILITY WITH AND WITHOUT LPD ERRORS

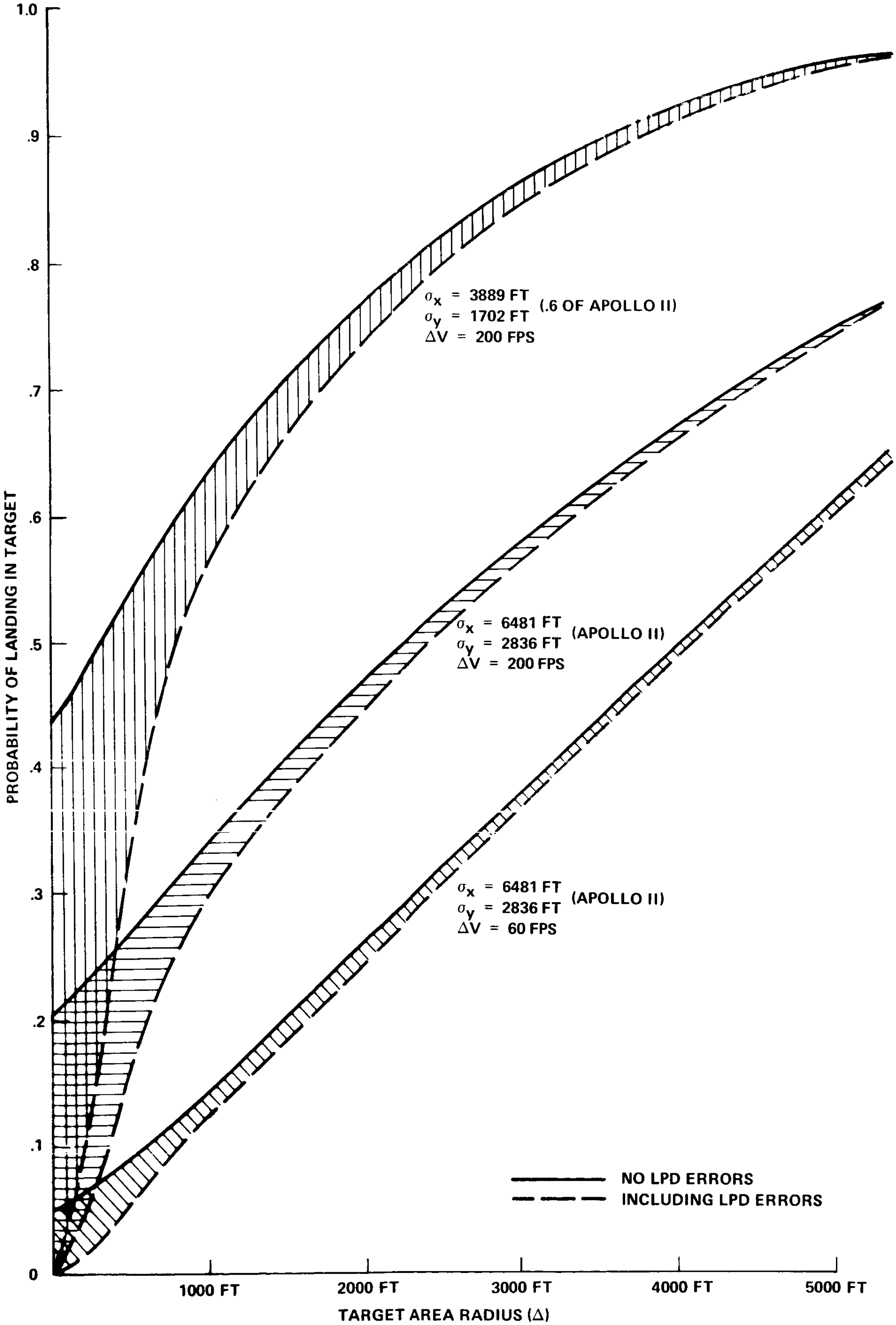


FIGURE 8a – PROBABILITY OF LANDING IN TARGET WITH AND WITHOUT LPD ERRORS

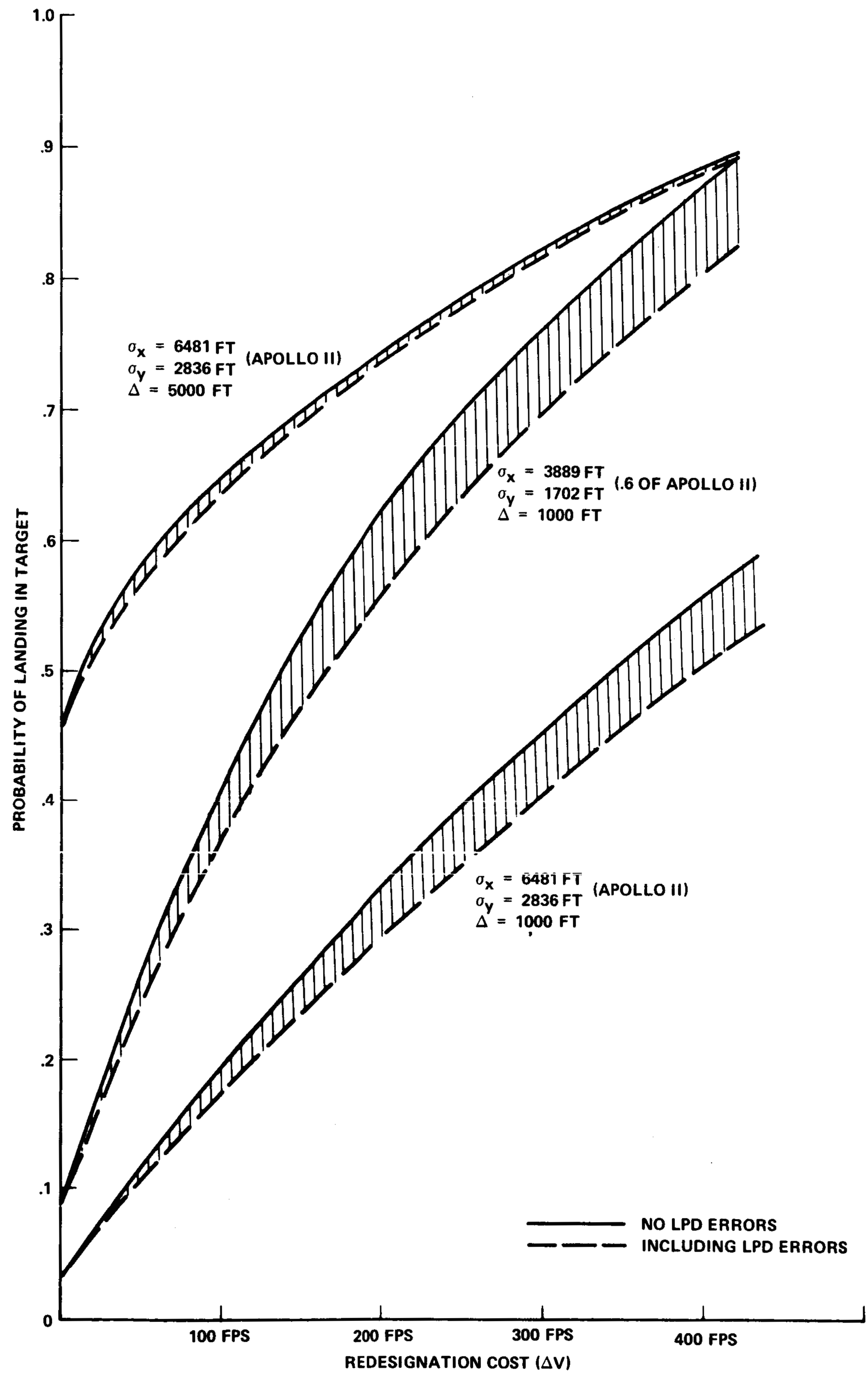


FIGURE 8b – PROBABILITY OF LANDING IN TARGET WITH AND WITHOUT LPD ERRORS

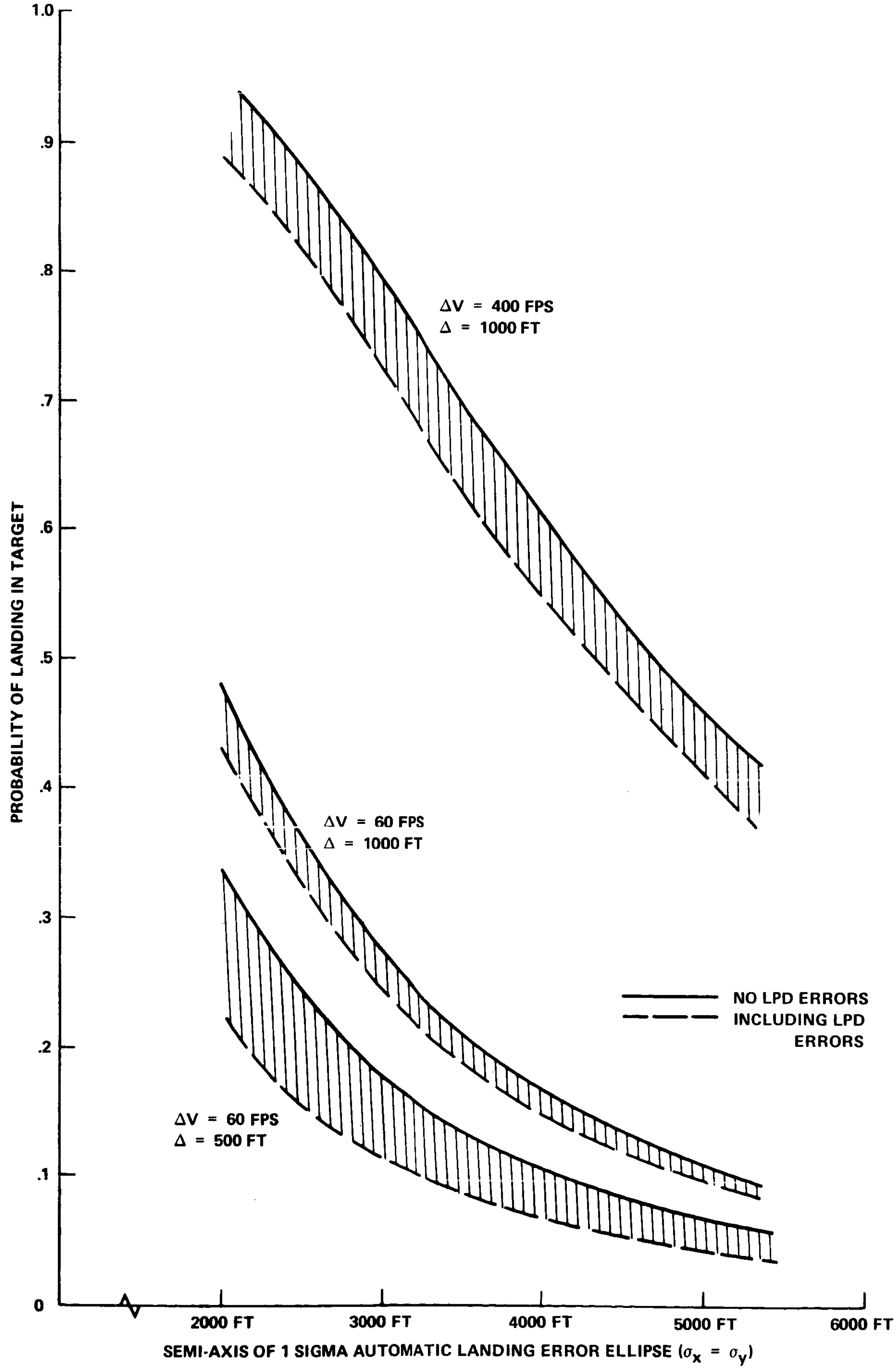


FIGURE 8c – PROBABILITY OF LANDING IN TARGET WITH AND WITHOUT LPD ERRORS

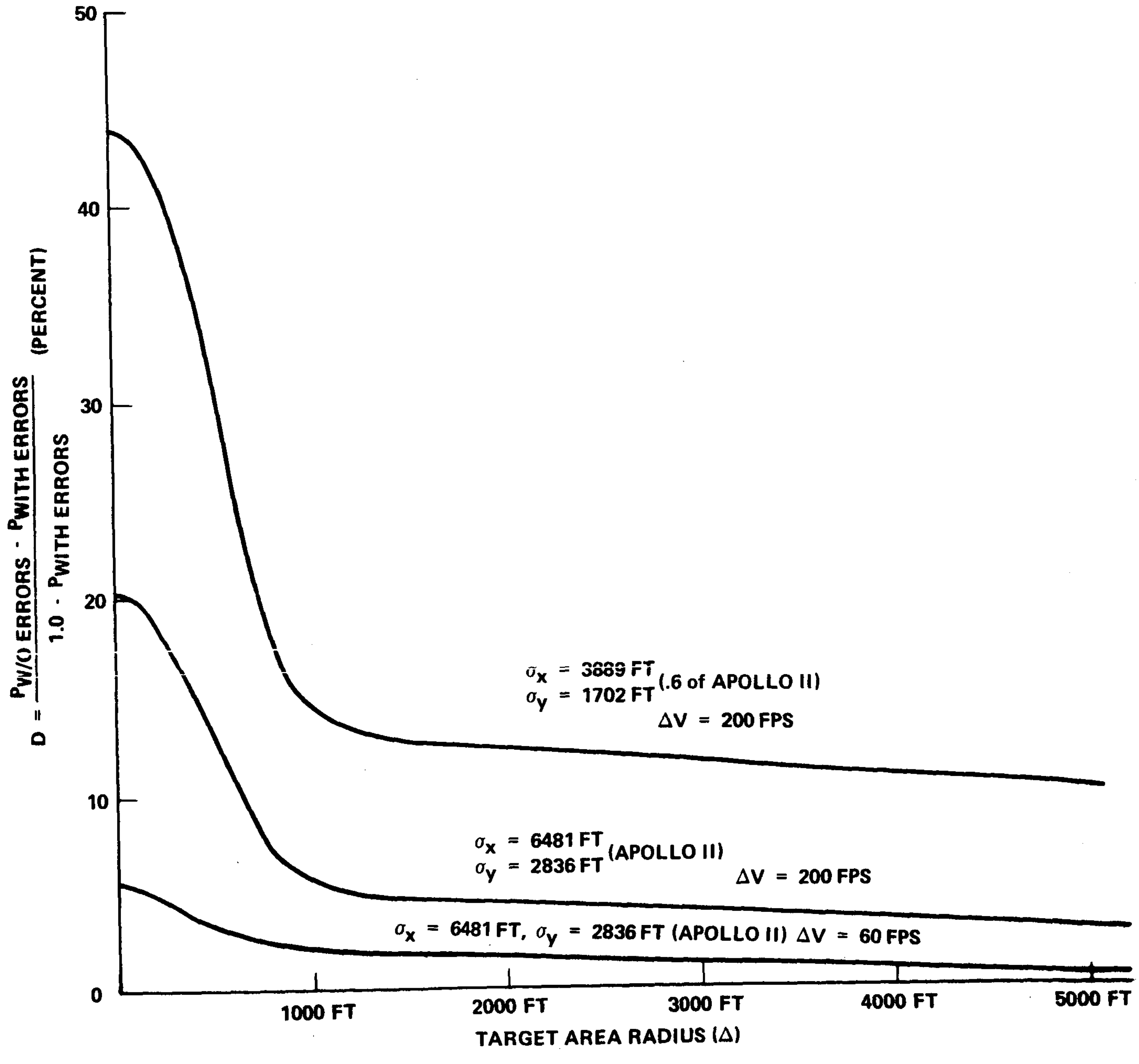


FIGURE 9a – ALTERNATE MEASURE OF LPD ERROR EFFECTS, D, VS. TARGET AREA RADIUS

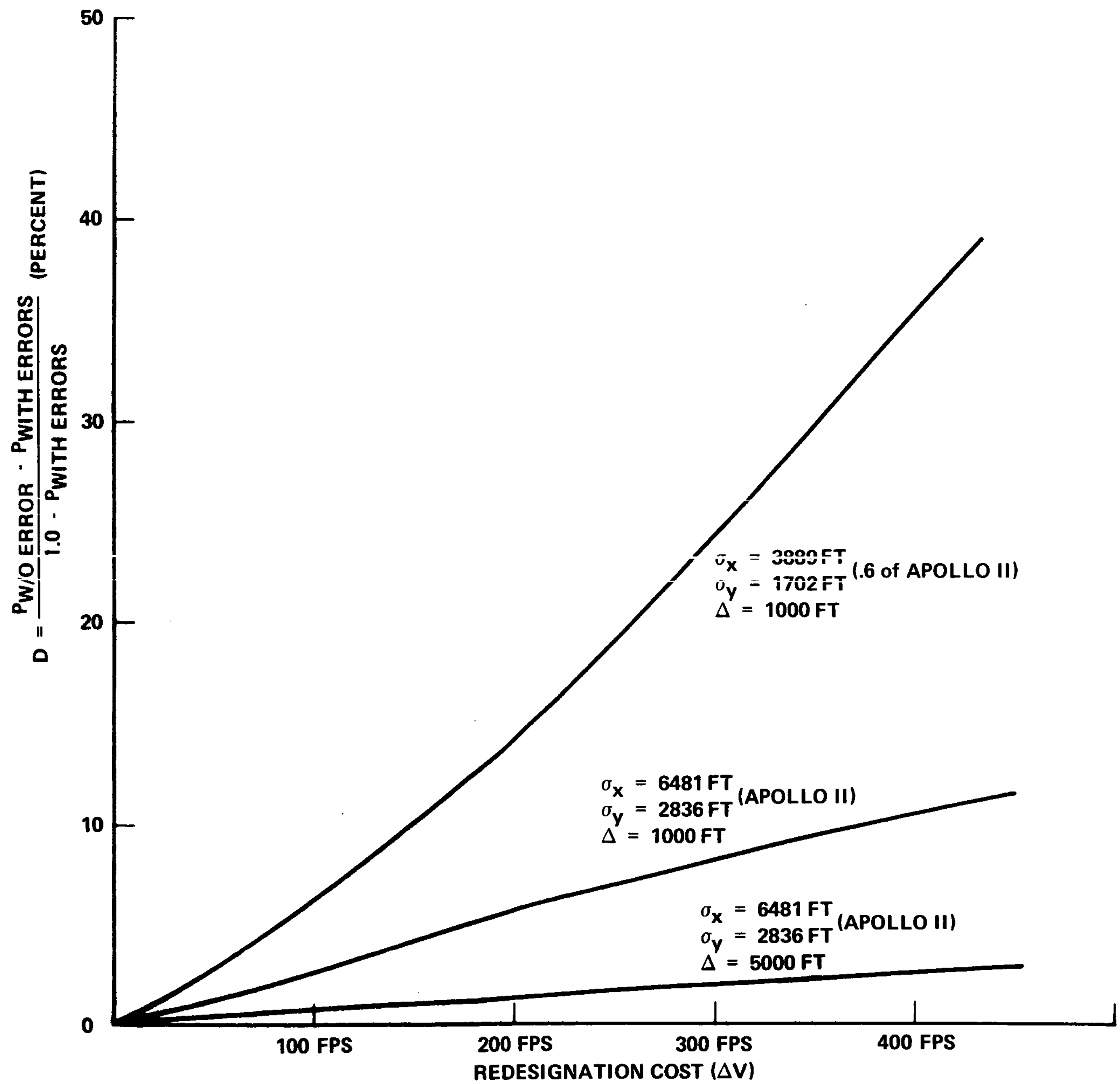


FIGURE 9b – ALTERNATE MEASURE OF LPD ERROR EFFECTS, D, VS. REDESIGNATION COST

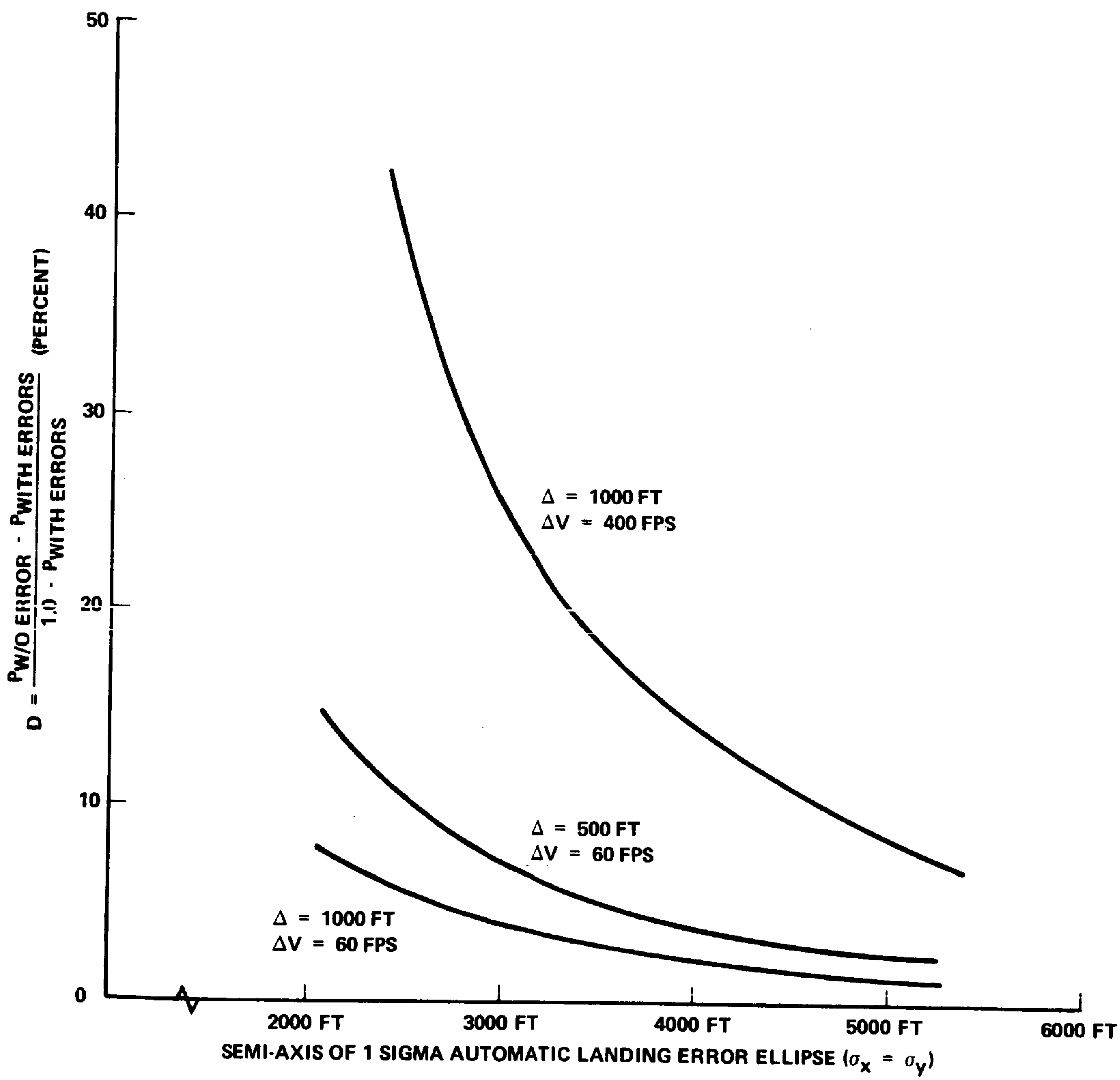


FIGURE 9c – ALTERNATE MEASURE OF LPD ERROR EFFECTS, D, VS. ERROR ELLIPSE SIZE

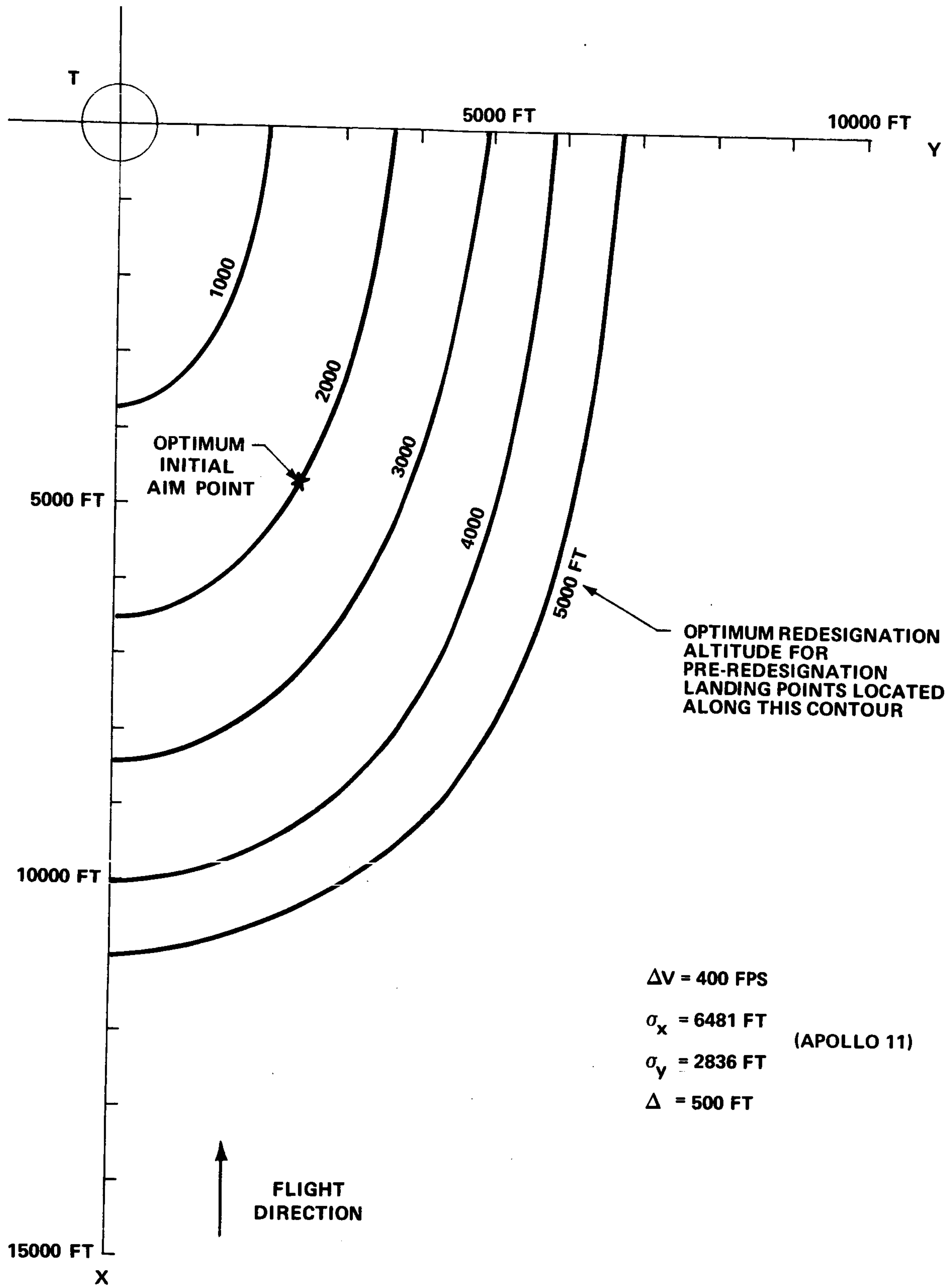
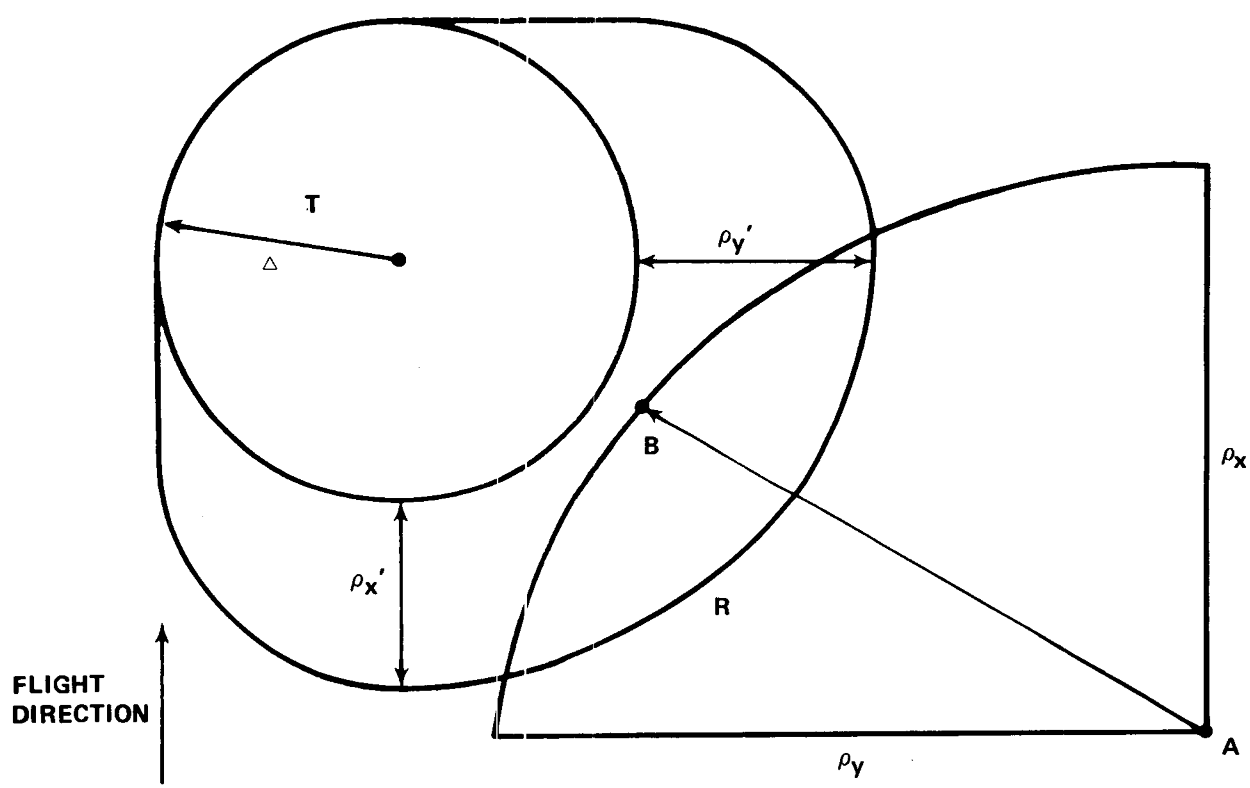


FIGURE 10 - SINGLE REDESIGNATION AT OPTIMUM ALTITUDE



ΔV = REDESIGNATION ΔV BUDGET
ρ_x = DOWNRANGE REDESIGNATION CAPABILITY OF $K \cdot \Delta V$ AT 5000 FT. ALTITUDE
ρ_y = CROSSRANGE REDESIGNATION CAPABILITY OF $K \cdot \Delta V$ AT 5000 FT. ALTITUDE
ρ_x' = DOWNRANGE REDESIGNATION CAPABILITY OF $(1\text{-}K) \cdot \Delta V$ AT 1000 FT. ALTITUDE
ρ_y' = CROSSRANGE REDESIGNATION CAPABILITY OF $(1\text{-}K) \cdot \Delta V$ AT 1000 FT. ALTITUDE

FIGURE 11 – DOUBLE REDESIGNATION PROCEDURE

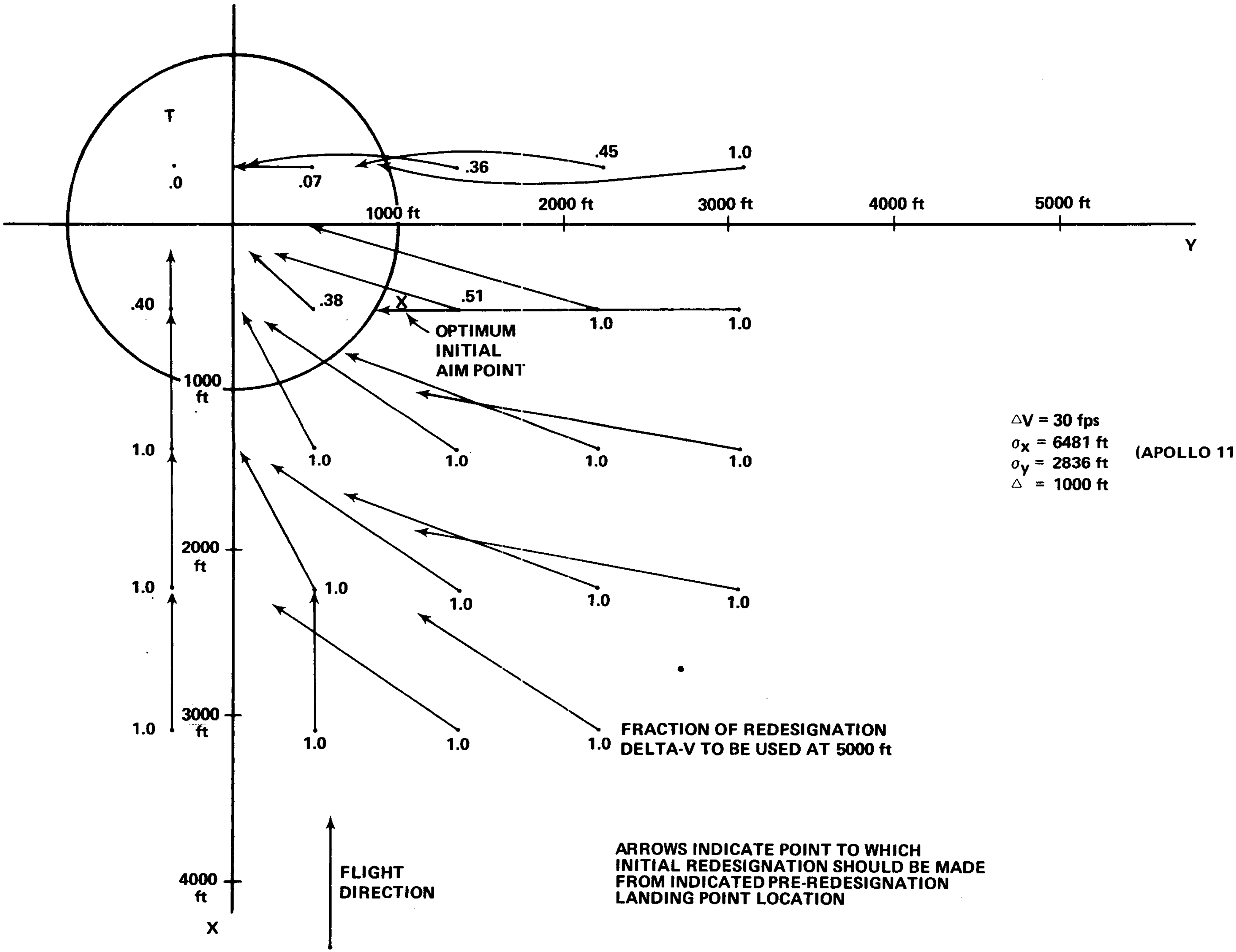


FIGURE 12 – OPTIMUM DOUBLE REDESIGNATION PROCEDURE

	SINGLE REDESIGNATION MADE AT 5000 FT. ALTITUDE WITHOUT ERROR	DOUBLE REDESIGNATION (AT 5000 FT. AND 1000 FT.) INCLUDING LPD ERRORS	SINGLE REDESIGNATION AT OPTIMUM ALTITUDE INCLUDING LPD ERRORS	SINGLE REDESIGNATION AT 5000 FT. INCLUDING LPD ERRORS
Δ = 500 FT. ΔV = 400 FPS $\sigma_x = \sigma_y$ = 1640 FT.	97.9%	91.7%	89.7%	60.7%
Δ = 500 FT. ΔV = 200 FPS $\sigma_x = \sigma_y$ = 1640FT.	88.1%	70.2%	69.5%	54.6%
Δ = 1000 FT. ΔV = 200 FPS $\sigma_x = \sigma_y$ = 1640 FT.	94.0%	91.5%	90.8%	86.2%
Δ = 1000 FT. ΔV = 200 FPS $\sigma_x = \sigma_y$ = 2500 FT.	72.0%	68.7%	66.9%	64.2%

FIGURE 13 - PROBABILITY OF LANDING IN A CIRCLE OF RADIUS Δ FOR DIFFERENT REDESIGNATION PROCEDURES